ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 258 - Introduction to Differential Equations

Final EXAMINATION

30.05.2017

STUDENT NUMBER:

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 110 minutes

ANSWER KEY

Question	Grade	Out of
1		10
2		20
3		20
4		20
5		20
6		20
Total		110

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. Solve the following integro-differential equation

$$y' = \sin x + \int_0^x y(x-u)\cos u\,du, \qquad y(0) = 0.$$

$$\mathcal{L}\{y'\} = \mathcal{L}\{\sin x + y(x) * \cos x\}$$

$$sY(s) - \cancel{y(0)}^{0} = \mathcal{L}\{\sin x\} + \mathcal{L}\{y\}\mathcal{L}\{\cos x\}$$

$$sY(s) = \frac{1}{s^2+1} + \frac{s}{s^2+1}Y(s)$$

$$\left(s - \frac{s}{s^2+1}\right)Y(s) = \frac{1}{s^2+1}$$

$$\frac{s^3+s-s}{s^2+1}Y(s) = \frac{1}{s^2+1} \quad \Rightarrow \quad Y(s) = \frac{1}{s^3}$$

$$y(x) = \mathcal{L}^{-1}\left\{\frac{1}{s^3}\right\} = \frac{x^2}{2}$$

Question 2. Use Laplace transforms to solve the following IVP

$$y'' + 2y' + 5y = \begin{cases} 1, & 0 < x < 2, \\ 0, & x > 2, \end{cases} \qquad y(0) = y'(0) = 0.$$

$y'' + 2y' + 5y = 1 - u(x-2)$

$\mathcal{L}\{y'' + 2y' + 5y\} = \mathcal{L}\{1 - u(x-2)\} \Rightarrow$

$s^2 Y(s) - s y(0) - y'(0) + 2[sY(s) - y(0)] + 5Y(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$

$(s^2 + 2s + 5) Y(s) = \frac{1}{s} - \frac{e^{-2s}}{s}$

$Y(s) = \frac{1}{s(s^2+2s+5)} - \frac{e^{-2s}}{s(s^2+2s+5)}$

$\frac{1}{s(s^2+2s+5)} = \frac{A}{s} + \frac{Bs+C}{s^2+2s+5} = \frac{(A+B)s^2 + (2A+C)s + 5A}{s(s^2+2s+5)}$

$A + B = 0, \quad 2A + C = 0; \quad 5A = 1 \quad$ so $\quad A = \frac{1}{5} \quad B = -\frac{1}{5} \quad C = -\frac{2}{5}$

$\frac{1}{s(s^2+2s+5)} = \frac{\frac{1}{5}}{s} + \frac{-\frac{1}{5}s - \frac{2}{5}}{s^2+2s+5} = \frac{\frac{1}{5}}{s} - \frac{1}{5}\frac{s+2}{s^2+2s+5} = \frac{\frac{1}{5}}{s} - \frac{1}{5}\frac{s+1}{(s+1)^2+4} - \frac{1}{10}\frac{2}{(s+1)^2+4}$

$\mathcal{L}^{-1}\left\{\frac{1}{s(s^2+2s+5)}\right\} = \frac{1}{5} - \frac{1}{5}e^{-x}\cos 2x - \frac{1}{10}e^{-x}\sin 2x$

$y(x) = \frac{1}{5} - \frac{1}{5}e^{-x}\cos 2x - \frac{1}{10}e^{-x}\sin 2x - \left[\frac{1}{5} - \frac{1}{5}e^{-x+2}\cos(2x-4) - \frac{1}{10}e^{-x+2}\sin(2x-4)\right]u(x-$