MATH 258

Exercise Set

1) Solve the following equations using Laplace transforms

a- $y'' + 4y = \begin{cases} \sin x & 0 \le x \le 2\pi \\ 0 & x \geq 2\pi \end{cases}$ $\quad y(0) = 1 ; y'(0) = 3$

b- $y'' + 5y' + 6y = \begin{cases} 0 & 0 \le x < 1 \\ x & 1 < x < 5 \\ 1 & x > 5 \end{cases}$ $\quad y(0) = 0 ; y'(0) = 2$

c- $y'' + 3y' + y = \begin{cases} e^{-x} & 0 \le x < 3 \\ 1 & x > 3 \end{cases}$ $\quad y(0) = 2 ; y'(0) = 1$

2) Using convolution, find the inverse of

a) $\dfrac{s}{(s^2+1)^2}$ b) $\dfrac{1}{s^3(s^2+1)}$ c) $\dfrac{s+1}{(s^2+1)^2}$

3) Solve the following integral equations or integro-differential equations

a) $y(x) + \int_0^x (x-v)\, y(v)\, dv = x^2$

b) $y' + y - \int_0^x y(v) \sin(x-v)\, dv = -\sin x \; ; \quad y(0) = 1.$

c) $y' + \int_0^x (x-v)\, y(v)\, dv = x \quad y(0) = 0$

4) Find the first four nonzero terms in a power series expansion about $x=0$ for a general solution to the given ODE.

a) $y'' + (x-1)y' + y = 0$

b) $(2x-3)y'' - xy' + y = 0$

c) $(x^2+2)y'' + 2xy + 3y = 0 \; ; \quad y(0) = 1 ; y'(0) = 2$

5) Find the Fourier series associated with the following functions

a) $f(x) = x + x^2 \quad -\pi < x < \pi$

b) $f(x) = x + |x| \quad -\pi < x < \pi$

c) $f(x) = e^x \quad -\pi < x < \pi$

d) $f(x) = \begin{cases} -1-x & -1 < x < 0 \\ x & 0 < x < 1 \end{cases}$

6) Find the Fourier cosine and sine series of

a) $f(x) = x^3 \quad 0 < x < \pi$

b) $f(x) = \cos 2x \quad 0 < x < \pi$

c) $f(x) = \begin{cases} 0 & 0 < x < \frac{1}{2} \\ 1 & \frac{1}{2} < x < 1 \end{cases}$

d) $f(x) = \begin{cases} x & 0 < x < 2 \\ 2-x & 2 < x < 4 \end{cases}$

7) Solve the following partial differential equations.

a) $u_t = u_{xx} \quad 0 < x < \pi \,;\ t > 0$

$u(0,t) = u(\pi,t) = 0 \quad t > 0 \,;\quad u(x,0) = \sin \pi x \quad 0 < x < \pi.$

b) $u_t = 2u_{xx} \quad 0 < x < 1$

$u(0,t) = u(1,t) = 0 \quad t > 0 \,;\quad u(x,0) = x(1-x) \quad 0 < x < 1.$

c) $u_t = u_{xx} \,; \quad 0 < x < 1 \,,\ t > 0$

$u(0,t) = u(1,t) = 1 \quad t > 0$

$u(x,0) = \cos \pi x \quad 0 < x < \pi$

e) $u_t = u_{xx} \quad 0 < x < 1 \quad t > 0$

$u(0,t) = u(1,t) = 0 \quad t > 0$

$u(x,0) = \cos(\pi x) \quad 0 < x < \pi$

d) $u_t = 10 u_{xx} \quad 0 < x < L \,,\ t > 0$

$u(0,t) = 0 \,;\quad u(L,t) = T \quad t > 0$

$u(x,0) = \begin{cases} 0 & 0 < x < \frac{L}{2} \\ T & \frac{L}{2} < x < L \end{cases}$