ÇANKAYA UNIVERSITY

Department of Mathematics

MATH 258 - Introduction to Differential Equations

FIRST MIDTERM EXAMINATION

20.03.2017

STUDENT NUMBER: ANSWER KEY

NAME-SURNAME:

SIGNATURE:

INSTRUCTOR:

DURATION: 100 minutes

Question	Grade	Out of
1		17
2		17
3		17
4		17
5		17
6		17
Total		102

IMPORTANT NOTES:

1) Please make sure that you have written your student number and name above.

2) Check that the exam paper contains 6 problems.

3) Show all your work. No points will be given to correct answers without reasonable work.

Question 1. Solve the equation $2 + \frac{dx}{dy} = y - x$.

I. $\frac{dx}{dy} = \underbrace{y-x-2}_{G(y-x)}$ $\quad y-x=u$ $\quad 1-\frac{dx}{dy}=\frac{du}{dy}$ $\quad \frac{dx}{dy}=1-\frac{du}{dy}$ $\quad 1-\frac{du}{dy}=u-2$

$3-u=\frac{du}{dy}$ $\quad \frac{du}{3-u}=dy$ $\quad -\ln|3-u|=y+\ln c_1$ $\quad \frac{1}{3-u}=\tilde{c}_1 e^y$ $\quad c=(3-u)e^y$

$\boxed{(3-y+x)e^y=c}$

II. $\underbrace{1}_{M(x,y)}\cdot dx - \underbrace{(y-x-2)}_{N(x,y)}dy=0$ $\quad M_y=0\neq N_x=1$ (not exact)

$\frac{N_x-M_y}{M}=1=f(y)$ $\quad \frac{d\mu}{\mu}=dy$ $\quad \ln|\mu|=y+\ln c$ $\quad \mu(y)=e^y$

$e^y dy - e^y(y-x-2)dy=0$ (exact)

$F_x=e^y$

$F_y=-ye^y+xe^y+2e^y$

$\begin{cases} F(x,y)=xe^y+\varphi(y) \\ xe^y+\varphi'(y)=-ye^y+xe^y+2e^y \end{cases}$ $\quad \varphi'(y)=-ye^y+2e^y$

$\varphi(y)=-\int y\,d(e^y)+2e^y+c_1$ $\quad \varphi(y)=-\left[ye^y-\int e^y dy\right]+2e^y+c_1$

$\varphi(y)=-ye^y+\underbrace{e^y+2e^y}+c_1$ $\quad F(x,y)=xe^y-ye^y+3e^y+c_1$

$xe^y-ye^y+3e^y=c$ $\quad \boxed{(3-y+x)e^y=c}$

III. $\frac{dy}{dx}+\underbrace{1}_{P(y)}\cdot x=\underbrace{y-2}_{Q(y)}$ (linear) $\quad \mu(y)=e^{\int dy}=e^y$

$e^y\cdot x=\int e^y(y-2)dy=\int(y-2)d(e^y)=(y-2)e^y-\int e^y dy=(y-2)e^y-e^y+c$

$x=y-2-1+ce^{-y}$ $\quad (x-y+3)=ce^{-y}$ $\quad \boxed{(x-y+3)e^y=c}$

Question 2. It is known that the equation

$$(asiny + y^2e^x)dx + (xcosy + bye^x)dy = 0$$

is exact. Find a and b and then solve the equation.

$\underbrace{(asiny + y^2e^x)}_{M(x,y)}dx + \underbrace{(xcosy + bye^x)}_{N(x,y)}dy = 0$ (exact)

$M_y = acosy + 2ye^x = N_x = cosy + bye^x \Rightarrow a = 1,\ b = 2$

$(siny + y^2e^x)dx + (xcosy + 2ye^x)dy = 0$ (exact)

$$\begin{cases} F_x = siny + y^2e^x \\ F_y = xcosy + 2ye^x \end{cases} \quad \begin{cases} F(x,y) = xsiny + y^2e^x + \varphi(y) \\ xcosy + 2ye^x + \varphi'(y) = xcosy + 2ye^x \Rightarrow \varphi(y) = c_1 \end{cases}$$

$$\boxed{xsiny + y^2e^x = c}$$