



MATH 258 - Introduction to Differential Equations

Review Exercises

01.12.2016

1) Find a general solution to given differential equations.

a) $y'' - y' - 2y = 0$.

Ans. $y(x) = c_1 e^{2x} + c_2 e^{-x}$

b) $4w'' + 20w' + 25w = 0$.

Ans. $w(x) = c_1 e^{-5x/2} + c_2 x e^{-5x/2}$

c) $z'' - 4z' + 7z = 0$

Ans. $z(x) = c_1 e^{2x} \cos(\sqrt{3}x) + c_2 e^{2x} \sin(\sqrt{3}x)$

d) $y''' + y'' + 3y' - 5y = 0$

Ans. $y(x) = c_1 e^x + c_2 e^{-x} \cos(2x) + c_3 e^{-x} \sin(2x)$

2) Solve the given IVP's

a) $y'' - 4y' - 5y = 0$, $y(-1) = 3$ and $y'(-3) = 9$.

Ans. $y(x) = 2e^{5(x+1)} + e^{-(x+1)}$

b) $y'' + 2y' + y = 0$, $y(0) = 1$ and $y'(0) = -3$.

Ans. $y(x) = e^{-x} - 2xe^{-x}$

c) $y''' - 4y'' + 7y' - 6y = 0$, $y(0) = 1$, $y'(0) = 0$ and $y''(0) = 0$.

Ans. $y(x) = e^{2x} - \sqrt{2}e^x \sin(\sqrt{2}x)$

3) Find a particular solution to given differential equations.

a) $4y'' + 11y' - 3y = -2xe^{-3x}$.

Ans. $y_p(x) = \left(\frac{x}{13} + \frac{8}{169}\right)xe^{-3x}$

b) $y'' + 2y' + 4y = 111e^{2x} \cos(3x)$.

Ans. $y_p(x) = e^{2x} \cos(3x) + 6e^{2x} \sin(3x)$

c) $y''' - y'' + y = \sin x$

Ans. $y_p(x) = \frac{1}{5} \cos x + \frac{2}{5} \sin x$

d) $y''' + y'' - 2y = xe^x$

Ans. $y_p(x) = \frac{1}{10}x^2e^x - \frac{4}{25}xe^x$

4) Find the solution to the given IVP's.

a) $y'' - y = \sin x - e^{2x}$, $y(0) = 1$ and $y'(0) = -1$.

Ans. $y(x) = -\frac{1}{2} \sin x - \frac{1}{3} e^{2x} + \frac{3}{4} e^x + \frac{7}{12} e^{-x}$

b) $y' - y = 1$, $y(0) = 0$.

Ans. $y(x) = e^x - 1$

c) $y'' + y = 2e^{-x}$, $y(0) = 0$ and $y'(0) = 0$.

Ans. $y(x) = e^{-x} - \cos x + \sin x$

5) Find a general solution to given differential equations.

a) $y'' + y = \sec x$.

Ans. $y(x) = \cos x \ln |\cos x| + x \sin x + c_1 \cos x + c_2 \sin x$

b) $y'' + 4y' + 4y = e^{-2x} \ln x$.

Ans. $y(x) = \frac{(2 \ln x - 3) x^2 e^{-2x}}{4} + c_1 e^{-2x} + c_2 x e^{-2x}$

c) $y'' + y = \tan^2 x$.

Ans. $y(x) = \sin x \ln |\sec x + \tan x| - 2 + c_1 \cos x + c_2 \sin x$

6) Find a general solution to given differential equations for $x > 0$.

a) $x^2 y'' + 2xy' - 6y = 0$.

Ans. $y(x) = c_1 x^{-3} + c_2 x^2$

b) $y'' + \frac{6}{x} y' + \frac{4}{x^2} y = 0$.

Ans. $y(x) = c_1 x^{-1} + c_2 x^{-4}$

c) $x^2 y'' + xy' + 9y = -\tan(3 \ln x)$.

Ans. $y(x) = c_1 \cos(3 \ln x) + c_2 \sin(3 \ln x) + \frac{1}{9} \cos(3 \ln x) \ln |\sec(3 \ln x) + \tan(3 \ln x)|$

d) $x^2 y'' - xy' + y = x \left(1 + \frac{3}{\ln x}\right)$.

Ans. $y(x) = c_1 x + c_2 x \ln x + \frac{1}{2} x (\ln x)^2 + 3x \ln x (\ln |\ln x|)$

7) Given that $y_1(x) = x$ is a solution to

$$y'' - \frac{1}{x} y' + \frac{1}{x^2} y = 0,$$

use the reduction of order procedure to determine a second linearly independent solution for $x > 0$.

Ans. $y_2(x) = x \ln x$

8) Given that $y_1(x) = x + 1$ is a solution to

$$xy'' - (x + 1)y' + y = x^2,$$

find the general solution.

Ans. $y = c_1 e^x + c_2(1 + x) - x^2$

9) Given that $y_1(x) = e^{-5x}$ is a solution to

$$xy'' + (5x - 1)y' - 5y = x^2 e^{-5x},$$

find the general solution.

Ans. $y = c_1(5x - 1) + c_2 e^{-5x} - \frac{x^2 e^{-5x}}{10}$

10) Given that $y_1(x) = \frac{1}{x}$ is a solution to

$$x^2 y'' - 2xy' - 4y = 0,$$

find the general solution.

Ans. $y = c_1 x^4 + c_2 \frac{1}{x}$