Çankaya University
Department of Mathematics and Computer Science

MCS 258 - Exercise Sheet I

July 1, 2015

In problems 1-5, solve the differential equation.

1. $\dfrac{dy}{dx} = \dfrac{x+3y}{x-y}$

2. a) $xy' - y = x^2 e^{-x}$ b) $xy' + 2y = 3xe^{x^3}$

3. $e^{x^2} \sec y dx + \dfrac{1}{x} \sin y dy = 0$, $y(1) = 0$

4. a) $\dfrac{dy}{dx} - \dfrac{y}{3x} = 2x^4 y^4$ b) $y' - \dfrac{3}{x} y = x^4 \sqrt[3]{y}$

5. a) $(y-x)dx + (y+x)dy = 0$

b)$(2xy + e^y)dx + (x^2 + xe^y)dy = 0$

c)$(3x^2 \tan y - \dfrac{2y^3}{x^3})dx + (x^3 \sec^2 y + 4y^3 + \dfrac{3y^2}{x^2})dy = 0$

6. Find a suitable integrating factors that make each of the following equations exact.

a)$(2xy^2 - 3y^3)dx + (7 - 3xy^2)dy = 0$

b)$(2y^2)dx + (2x + 3xy)dy = 0$

7. Reduce the following Bernoulli Equation,

$$(x^2 - 1)\frac{dy}{dx} - 9x^{3/2}y = (x^2+1)y^{8/9},\ x > 1$$

to a linear equation. (Do not solve the equation.)

8. Consider the non-exact equation,

$$x(y^2+1)dx + (1 - \frac{x^2}{2})ydy = 0,\ x, y > 0$$

a) Show that $\mu(x) = \dfrac{1}{(y^2+1)^{3/2}}$ is an integrating factor.

b)Using this integrating factor solve the given differential equation.

9. Using the substitution $\vartheta = x^2 y - 2$, find the general solution of the following differential equation,

$$x^2 y' + 2xy = x\sqrt{x^2 y - 2}.$$

10. a)Explain why the IVP,

$$y' + \frac{1}{x}y = xy^{-2}, \; y(1) = 2$$

has a unique solution.

b) Solve the given IVP.

11. In each of the following problems, find an integrating factor and the general solutions of them,

a) $(1 - x^2y)dx + x^2(y - x)dy = 0.$

b) $\cos x dx + (y + \sin y + \sin x)dy = 0.$

12. Find an integrating factor of the form $x^m y^n$ and solve the following differential equation,

$$(4x + 3y^2)dx + 2xydy = 0$$

13. a) Verify that the differential equation,

$$y^2dx + [xy - \tan(xy)]dy = 0$$

is not exact.

b) Show that $\frac{1}{y^2}\cos(xy)$ is an integrating factor for given differential equation in **a)**.

c) Using this integrating factor, solve the given differential equation in **a)**.

14. Given equation

$$(y - xy^2)dx + (x + x^2y^2)dy = 0.$$

a) Show that $\frac{1}{x^2y^2}$, is an integrating factor for given differential equation.

b) Using this integrating factor, solve the given differential equation

15. Solve the following IVP,

$$xy' - y = x\tan(\frac{y}{x}), \; y(1) = \frac{\pi}{2}.$$