In Problems 1–20, determine the Laplace transform of the given function using Table 7.1 and the properties of the transform given in Table 7.2. [Hint: In Problems 12–20, use an appropriate trigonometric identity.]

1. $t^2 + e^t \sin 2t$
2. $3t^2 - e^{2t}$
3. $e^{-t}\cos 3t + e^{6t} - 1$
4. $3t^4 - 2t^2 + 1$
5. $2t^2e^{-t} - t + \cos 4t$
6. $e^{-2t}\sin 2t + e^{3t}t^2$
7. $(t-1)^4$
8. $(1 + e^{-t})^2$
9. $e^{-t}t \sin 2t$
10. $te^{2t}\cos 5t$
11. $\cosh bt$
12. $\sin 3t \cos 3t$
13. $\sin^2 t$
14. $e^{7t}\sin^2 t$
15. $\cos^3 t$
16. $t \sin^2 t$
17. $\sin 2t \sin 5t$
18. $\cos nt \cos mt$, $m \neq n$
19. $\cos nt \sin mt$, $m \neq n$
20. $t \sin 2t \sin 5t$
21. Given that $\mathscr{L}\{\cos bt\}(s) = s/(s^2 + b^2)$, use the translation property to compute $\mathscr{L}\{e^{at}\cos bt\}$.
22. Starting with the transform $\mathscr{L}\{1\}(s) = 1/s$, use formula (6) for the derivatives of the Laplace transform to show that $\mathscr{L}\{t\}(s) = 1/s^2$, $\mathscr{L}\{t^2\}(s) = 2!/s^3$, and, by using induction, that $\mathscr{L}\{t^n\}(s) = n!/s^{n+1}$, $n = 1, 2, \ldots$.
23. Use Theorem 4 to show how entry 32 follows from entry 31 in the Laplace transform table on the inside back cover of the text.
24. Show that $\mathscr{L}\{e^{at}t^n\}(s) = n!/(s-a)^{n+1}$ in two ways:
    (a) Use the translation property for $F(s)$.
    (b) Use formula (6) for the derivatives of the Laplace transform.
25. Use formula (6) to help determine
    (a) $\mathscr{L}\{t \cos bt\}$.
    (b) $\mathscr{L}\{t^2 \cos bt\}$.
26. Let $f(t)$ be piecewise continuous on $[0, \infty)$ and of exponential order.
    (a) Show that there exist constants K and α such that
    $$|f(t)| \le Ke^{\alpha t} \quad \text{for all } t \ge 0\ .$$
    (b) By using the definition of the transform and estimating the integral with the help of part (a), prove that
    $$\lim_{s\to\infty} \mathscr{L}\{f\}(s) = 0\ .$$
27. Let $f(t)$ be piecewise continuous on $[0, \infty)$ and of exponential order α and assume $\lim_{t\to 0^+}[f(t)/t]$ exists. Show that
    $$\mathscr{L}\left\{\frac{f(t)}{t}\right\}(s) = \int_s^\infty F(u)\,du\ ,$$

In Problems 1–10, determine the inverse Laplace transform of the given function.

1. $\dfrac{6}{(s-1)^4}$
2. $\dfrac{2}{s^2+4}$
3. $\dfrac{s+1}{s^2+2s+10}$
4. $\dfrac{4}{s^2+9}$
5. $\dfrac{1}{s^2+4s+8}$
6. $\dfrac{3}{(2s+5)^3}$
7. $\dfrac{2s+16}{s^2+4s+13}$
8. $\dfrac{1}{s^5}$
9. $\dfrac{3s-15}{2s^2-4s+10}$
10. $\dfrac{s-1}{2s^2+s+6}$

In Problems 11–20, determine the partial fraction expansions for the given rational function.

11. $\dfrac{s^2-26s-47}{(s-1)(s+2)(s+5)}$
12. $\dfrac{-s-7}{(s+1)(s-2)}$
13. $\dfrac{-2s^2-3s-2}{s(s+1)^2}$
14. $\dfrac{-8s^2-5s+9}{(s+1)(s^2-3s+2)}$
15. $\dfrac{8s-2s^2-14}{(s+1)(s^2-2s+5)}$
16. $\dfrac{-5s-36}{(s+2)(s^2+9)}$
17. $\dfrac{3s+5}{s(s^2+s-6)}$
18. $\dfrac{3s^2+5s+3}{s^4+s^3}$

In Problems 1–14, solve the given initial value problem using the method of Laplace transforms.

1. $y'' - 2y' + 5y = 0$;
$y(0) = 2$, $y'(0) = 4$

2. $y'' - y' - 2y = 0$;
$y(0) = -2$, $y'(0) = 5$

3. $y'' + 6y' + 9y = 0$;
$y(0) = -1$, $y'(0) = 6$

4. $y'' + 6y' + 5y = 12e^t$;
$y(0) = -1$, $y'(0) = 7$

5. $w'' + w = t^2 + 2$;
$w(0) = 1$, $w'(0) = -1$

6. $y'' - 4y' + 5y = 4e^{3t}$;
$y(0) = 2$, $y'(0) = 7$

7. $y'' - 7y' + 10y = 9\cos t + 7\sin t$;
$y(0) = 5$, $y'(0) = -4$

8. $y'' + 4y = 4t^2 - 4t + 10$;
$y(0) = 0$, $y'(0) = 3$

9. $z'' + 5z' - 6z = 21e^{t-1}$;
$z(1) = -1$, $z'(1) = 9$

10. $y'' - 4y = 4t - 8e^{-2t}$;
$y(0) = 0$, $y'(0) = 5$

11. $y'' - y = t - 2$; $y(2) = 3$, $y'(2) = 0$

12. $w'' - 2w' + w = 6t - 2$;
$w(-1) = 3$; $w'(-1) = 7$

13. $y'' - y' - 2y = -8\cos t - 2\sin t$;
$y(\pi/2) = 1$, $y'(\pi/2) = 0$

14. $y'' + y = t$; $y(\pi) = 0$, $y'(\pi) = 0$

In Problems 15–24, solve for $Y(s)$, the Laplace transform of the solution $y(t)$ to the given initial value problem.

15. $y'' - 3y' + 2y = \cos t$;
$y(0) = 0$, $y'(0) = -1$

16. $y'' + 6y = t^2 - 1$;
$y(0) = 0$, $y'(0) = -1$

17. $y'' + y' - y = t^3$;
$y(0) = 1$, $y'(0) = 0$

18. $y'' - 2y' - y = e^{2t} - e^t$;
$y(0) = 1$, $y'(0) = 3$

19. $y'' + 5y' - y = e^t - 1$;
$y(0) = 1$, $y'(0) = 1$

20. $y'' + 3y = t^3$; $y(0) = 0$, $y'(0) = 0$

21. $y'' - 2y' + y = \cos t - \sin t$;
$y(0) = 1$, $y'(0) = 3$