MATH 258

Exercise Set I

a) Solve the given equation or IVP

1) $y' = \tan\left(\frac{y}{x}\right) + \frac{y}{x}$

2) $(2x+y)\sin y\,dx + (x\sin y + 3\cos y)\,dy = 0$

3) $3\frac{dy}{dx} + \frac{y}{x} = -xy^4 \ln x$

4) $\frac{dy}{dx} + 4y = x^2 e^{-4x}$

5) $(x^2+y^2)\,dx + 2xy\,dy = 0$

6) $(\tan y - 2)\,dx + \left(x\sec^2 y + \frac{1}{y}\right)dy = 0$

7) $\frac{dy}{dx} = \frac{y+x}{3x-y}$

8) $dx + (1 - x\tan y)\,dy = 0$

9) $y' - y\cot x = \frac{y^3}{\sin x}$

10) $e^{2x}\tan y\, y' = x$

11) $(2xy + 4x^3y^2)\,dx + (3x^2 + 4x^4 y)\,dy = 0$

12) $y' + \frac{2y}{x} = -x^2 y^2 \cos x$

13) $y' = \frac{x+y}{x-y}$; $y(1) = 1$

14) $\frac{dy}{dx} = \left(y + y^{5/3}\right)\sin x$

15) $\frac{dy}{dx} = \frac{x+3y}{x-y}$

16) $xy' - y = x^2 e^{-x}$

17) $xy' + y = 3xe^x$

18) $e^{x^2}\sec y\,dx + \frac{1}{x}\sin y\,dy = 0 \;;\; y(1) = 0$

19) $\frac{dy}{dx} - \frac{y}{3x} = 2x^4 y^4$

20) $\frac{dx}{dy} - 3\frac{x}{y} = y^4 x^{1/3}$

21) $(2xy + e^y)\,dx + (x^2 + xe^y)\,dy = 0$

22) $2y^2 dx + (2x + 3xy)\,dy = 0$

23) $(4x + 3y^2)\,dx + 2xy\,dy = 0$

24) $(y - xy^2)\,dx + (x + x^2 y^3)\,dy = 0$

25) $(\tan y + \frac{y^3}{x})\,dx + (x\sec^2 y + \sec y \tan y + 2y\ln x)\,dy = 0;\; y(1) = \pi.$

26) $(2y - xe^x)\,dx + x\,dy = 0 \;;\; y(1) = 2.$

27) $2(x+y)\,dy - y\,dx = 0$

28) $3x^2 y^2\,dx + (2x^3 y + 2y - 1)\,dy = 0;\; y(1) = 1.$

29) $\frac{dr}{d\theta} + r\tan\theta = \sec\theta$

30) $y\frac{dx}{dy} + 2x = 5y^3$

31) $\frac{dy}{dx} = (1+y^2)\tan x \qquad y(0) = \sqrt{3}$

32) $\frac{1}{\theta}\frac{dz}{d\theta} = \frac{z\sin\theta}{1+z^2} \qquad z(\pi) = 1.$

33) $(x^2 - xy)dx + x^2 dy = 0$

34) $(1+x^2)dy + (2xy - \tan x)dx = 0$

35) $2y(e^{y^2} + x)dy = dx$

36) $(2xy - \sec^2 x)dx + (x^2 + 2y)dy = 0$

37) $(1 + e^x y + xe^x y)dx + (xe^x + 2)dy = 0$

38) $\frac{dy}{dx} = \sqrt{x+y} - 1$; 39) $y' = \sin(x-y)$

40) $(2x - y)dx + (4x + y - 3)dy = 0$

41) $(2x + y + 4)dx + (x - 2y + 2)dy = 0$

b) Find integrating factors of the form $x^m y^n$ and then solve the following equations

1) $(4x + 3y^2)dx + 2xy\,dy = 0$

2) $(3x^4 y^2 + 4x^2 y)dx + (2x^5 y + 3x^3)dy = 0$

c) If x^2 is a solution of $y' - y - \frac{2}{x^3}y^2 + x^2 = 0$, find the general solution of the given equation.

d) If e^x is a solution of $y' = e^x - e^{3x} + 2e^{2x}y - e^x y^2$, solve the IVP $y' = e^x - e^{3x} + 2e^{2x}y - e^x y^2$; $y(0) = \frac{3}{2}$

e) If $\cos x$ is a solution of $(1 - \sin x \cos x)y' = -y^2 \cos x + y - \sin x$

Solve the given equation.